PHILIPPE TRINH

SUMMARY OF RESEARCH WORK

OCIAM
MATHEMATICAL INSTITUTE
24-29 ST. GILES
OXFORD, OX1 3LB

TRINH@MATHS.OX.AC.UK
WWW.PTRINH.COM
+44 (1865) 270 509

Chronological Summary

2003-2004	Chromosomal Breakpoint Re-Use in Genome Sequence Rearrangement
2004-2005	Genomic Features in the Breakpoint Regions Between Syntenic Blocks
2005	Steady Solutions of a Coupled KdV System
2006-2007	Resonant Solutions of a Broad Class of Forced KdV Equations
2007-2010	Exponential Asymptotics and Gravity-Capillary Waves
2007-2010	The Existence and Non-Existence of Waveless Ships

This is an abridged version with descriptions of recent projects only!
The full summary can be found at www.ptrinh.com

Research Interests

A vast portion of current research on scientific phenomena lies on the boundaries between different physical theories or different mathematical models—classical and quantum, rays and waves, linear and nonlinear, laminar and turbulent, and so on. Mathematically, these boundaries are often posed as questions in physical (often singular) asymptotics—the study of limits. This is my area of interest.

In the broadest sense, my goal as an applied mathematician is to develop analytical and numerical methodologies for the study of ordinary and partial differential equations. In particular, I've taken a recent interest in problems in fluid mechanics which involve the *breakdown* of traditional asymptotic analysis, subsequently requiring the use of asymptotics *beyond all orders*.

For example, in the study of nonlinear free-surface flows, the relative intractability of the surface equations generally requires consideration of the solution as a perturbative expansion,

$$S = \sum_{n=0}^{\infty} \epsilon^n S_n,$$

which depends on the limit of some small parameter, ϵ tending to zero. But the limit $\epsilon \to 0$ is often singular and difficulties arise; the series S is *divergent* and consequently by the Stokes Phenomenon, exponentially small terms can suddenly appear and disappear in the analytic continuation of the problem. The body of methods that are developed for their study is called *exponential asymptotics* or asymptotics *beyond all orders*, and applications of these special techniques have been found in diverse problems, ranging from models of crystal growth and viscous fingering [1], to the onset of turbulence [2] and the study of free-surface flows [4].

The Existence & Non-Existence of Waveless Ships

Date:	2007-2010 (Ph.D.)
Supervised by:	Prof. Jonathan Chapman (Oxford) Prof. Jean-Marc Vanden-Broeck (UCL)

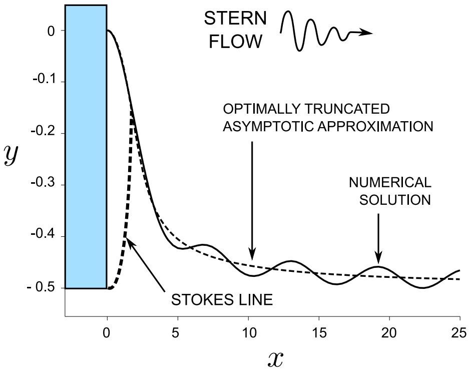

Figure 1: Asymptotic expansions in powers of the Froude number fail to capture waves present on the free surface. The key idea is that a Stokes line emerges from the corner singularity, across which an exponentially small wave turns on.

When an ideal fluid flows past a surface-piercing object or over an obstruction, waves are sometimes produced upstream or downstream of the disturbance. But in the low-speed limit, the traditional asymptotic series in powers of the Froude number fails to capture this phenomenon—this is the so-called *Low-Speed Paradox* first mentioned by Ogilvie [5]. It is now known that at low Froude numbers, the waves are in fact exponentially small and thus *beyond-all-orders* of regular asymptotics; their formation is a consequence of the divergence of the asymptotic series and the associated Stokes Phenomenon.

This underlying subtlety has been painfully problematic in regards to previous asymptotic and numerical treatments of the nonlinear ship-wave problem. In [6], Dagan and

Tulin showed that the analysis near a three-dimensional ship can be reduced to studying the two-dimensional ideal flow problem where the ship is modeled as a semi-infinite body with constant draft. This fully nonlinear free-surface problem was first computed by Vanden-Broeck and Tuck [7], and on the basis of numerical evidence, they conjectured that ship hulls with a single front face will always generate waves (such as in Figure 1).

Moreover, the earlier experimental work of Baba [10] had indicated that a bulbous bow can eliminate, or at least reduce the splash at the bow of a ship[1]. This prompted the discovery of seemingly waveless ships with bulbous profiles, first by Tuck and Vanden-Broeck [8] and later confirmed by Madurasinghe [11]—but again, only *numerically* so. Unfortunately, these results were refuted by the more comprehensive numerical study of Farrow and Tuck [9]; there, they had written that, "The free surface would at first sight appear to be waveless, but on closer examination of the numerical data, there are very small waves present" (see Figure 2).

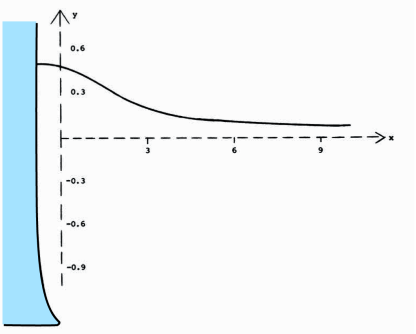

Figure 2: This seemingly waveless bulbous ship (their Figure 6) numerically discovered by Tuck and Vanden-Broeck [8] was later refuted by Farrow and Tuck [9]. Do there exist ship hulls which are effectively waveless?

Clearly, these are questions one cannot easily answer using simple numerics! Indeed, in *Reminiscences and Reflections: Ship Waves, 1950-2000* [12], Tulin asks:

> "The fundamental questions of whether such rising potential free-surface flows before bluff bodies exist...still remain open...Is it demonstrable...that continuous solutions will not exist in the limit of vanishing speed?...Do nonbreaking flows exist at all for surface-piercing ship forms of arbitrary form and thickness, at any speed?"

However, with the recent development of techniques in exponential asymptotics (see for example [4, 13]), we have been able to resolve most of these questions.

Our key result is the demonstration that the formation of waves near a ship is a necessary consequence of singularities in the ship's geometry or its analytic continuation, such as those corresponding to sharp corners. Afterwards, the use of complex-plane asymptotics, optimal truncation, and Stokes line smoothing can be applied to detect the Stokes lines emerging from each singularity, and then to derive the form of the exponential switched on as the line is crossed.

The analysis has been applied in order to prove that certain ship profiles will or will not produce a wake in the low-speed limit. For example, ships with an odd-number of corners can be shown to always generate a non-zero wake (in particular, this confirms the conjecture by Vanden-Broeck and Tuck [7] for the rectangular stern). Analytical criteria can also be derived for the existence of waveless ship forms (such as the ship in Figure 3).

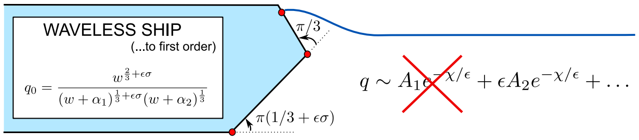

Figure 3: Our asymptotic analysis can be used to produce *almost* waveless ships. In this figure, we illustrate a ship with divergent corner-angles of approximately $\pi/3$ that can be made waveless to first order in ϵ for the correct choice of α_1, α_2, and σ. It has a bulbous shape, and the waves produced from each corner produces perfect phase cancellation (to first order).

The theory can also be extended to the study of ships that are not only piecewise linear, but also piecewise entire (in their analytic continuation); this includes the class of bulbous hulls previously studied by the aforementioned authors (see Figure 6).

Exponential Asymptotics & Gravity-Capillary Waves

Date:	2007-2010 (Ph.D.)
Supervised by:	Prof. Jonathan Chapman (Oxford)

It's well known that the study of water waves with both gravity and surface-tension present contains numerous difficulties not found when only one effect is included. For example, in the classical fishing rod experiment of Rayleigh [14], linearized theory predicts several regimes of interest for different values of the Froude and Bond numbers. However, the nonlinear structure of the problem is much more complex and many analytical and numerical studies have sought to unravel the full spectrum of solutions [e.g. 15, 16].

Much less is known on the topic of gravity-capillary flows past obstructions which cannot be considered small. For example, we could ask what is the effect of placing an order-one trapezoidal object on the bottom of a stream and how it differs from using a semi-circular object or a step. The standard technique is to linearise for small obstructions and thus the geometry of the obstruction is 'lost'. Instead, we propose that the low-Froude and low-Bond approximation can be used to elucidate analytical details of these problems—under this simplification, the geometry of the obstruction is preserved at the expense of dealing with asymptotic divergence. Hence this is a singular perturbation problem, Stokes lines can be expected, and techniques in expo-

---

[1] In the potential flow problem, a waveless solution past the stern (rear) of a ship is equivalent to a splashless solution at the bow (front) of a ship.

nential asymptotics must be used to observe the switching-on of waves (as illustrated in Figure 4).

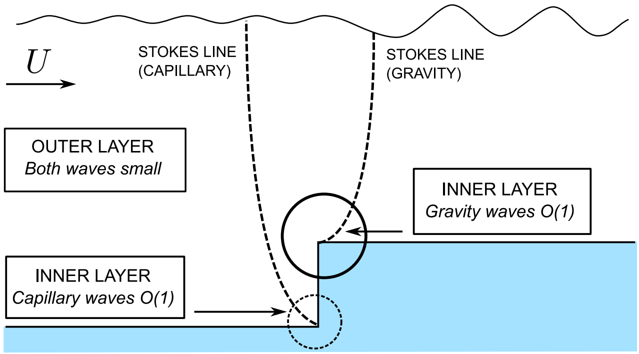

Figure 4: In the case of a rectangular step, there exist Stokes lines originating from the corner and stagnation points. There also exist additional turning points (not shown) corresponding to when the two exponentially-small waves are in phase; these may lie either on the bottom bed or on the free surface. As the Froude and Bond numbers are varied, different solutions are possible.

Here is an example of the novelty in this approach. When ideal fluid flow is considered over a step, a linearization of the free surface and the step height produces an approximation which replaces the step by a point source. As the Froude and Bond numbers tend to zero, there exist Stokes lines originating from the point and intersecting the free surface. If there are two *complex* wavenumbers to the linearized problem, then there are two symmetrical Stokes lines; if there are two *real* wavenumbers, then the two Stokes lines coalesce into one. Further analysis then tells us that in the former case, the free surface is of solitary type, while in the latter, there are capillary waves upstream and gravity waves downstream.

Now instead, if we preserve the nonlinearity of the step and take the limit as the Froude and Bond numbers tend to zero, the Stokes line structure changes dramatically. Now, there are Stokes lines emanating from the corner and stagnation point of the step (Fig. 4), as well as turning points (which may also produce Stokes lines) lying along either the boundary or the free surface. As the balance of the Froude and Bond numbers change, the Stokes line structure also changes, delineating regimes with different solutions. Even more subtle is what happens if the geometry of the obstruction is changed, say to an angled step. Now, a single singularity may in fact generate multiple *Stokes* lines.

There is thus a rich variety of possibilities of changing the Stokes-line structure and thus the range of possible solutions, all depending on the choice of the bottom topography as well as the crucial balance between Froude and Bond numbers. These possibilities have, in the past, *gone undetected* by regular linear or weakly nonlinear theories.

In general, our research has led to (1) a theoretical discovery of new gravity-capillary waves in the low-Froude, low-Bond limit, (2) a complete classification of such solutions, and (3) a development of the necessary exponential asymptotics for their description.

Resonant Solutions of Korteweg-de Vries Type Equations

Date:	2006-2007 (M.Sc.)
Supervised by:	Prof. Dave Amundsen (Carleton)

The Korteweg-de Vries (KdV) equation was originally motivated by the problem of modeling nonlinear waves in shallow water, but in recent years, generalizations and variations of the original KdV have been proposed in conjunction with other physical systems. For example, consider

$$u_t - \gamma u_{xxx} + \Delta u_x + \alpha u u_x + \beta u^2 u_x - \mu u_{xx} = f(x),$$

where the coefficients are physically-motivated constants. The equation represents a more general periodically forced and damped extended KdV (eKdV) equation, which includes special cases of modified KdV (mKdV, $\alpha = 0$), and regular KdV ($\alpha = 0$) equations. Each KdV-type equation has been associated with a wide range of physical applications, from shallow water in a tank subjected to forcing at one end [19] to transcritical flow of a stratified fluid [20].

In problems where both nonlinear and forcing effects are strong, it's known that a rich array of steady solutions emerge. And although analytic studies in this regime had been performed, they remained limited in their scope. However in [26], Amundsen, Cox, and Mortell developed a general framework based on singular perturbation and asymptotic matching in the context of the forced KdV. Our work extended these techniques to the case of the mKdV and eKdV equations, thus providing a more global perspective of the similarities and differences between the various KdV formulations.

In fact, our work provided a unification of the various KdV-type equations through an elegant asymptotic methodology; this produced a clear interpretation of general eKdV solutions as combinations of KdV and mKdV-like characteristics. Moreover, the resonant response of these equations was explored using AUTO (Figure 5) and approximations to the solutions, as well as the critical transitions between the various KdV-type equations were derived.

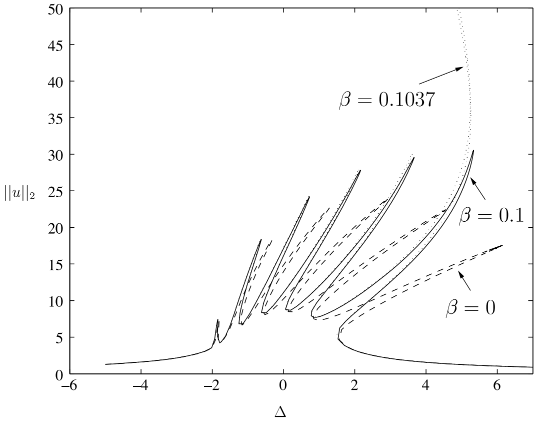

Figure 5: Resonant response for the eKdV equation with dispersion $\gamma = .005$, damping $\mu = .0015$, forcing $f(x) = \cos \pi x$, quadratic nonlinearity $\alpha = 2$, as the cubic nonlinearity β is increased. Near the value $\beta \approx 0.1037$, a homoclinic orbit turns into a heteroclinic orbit.

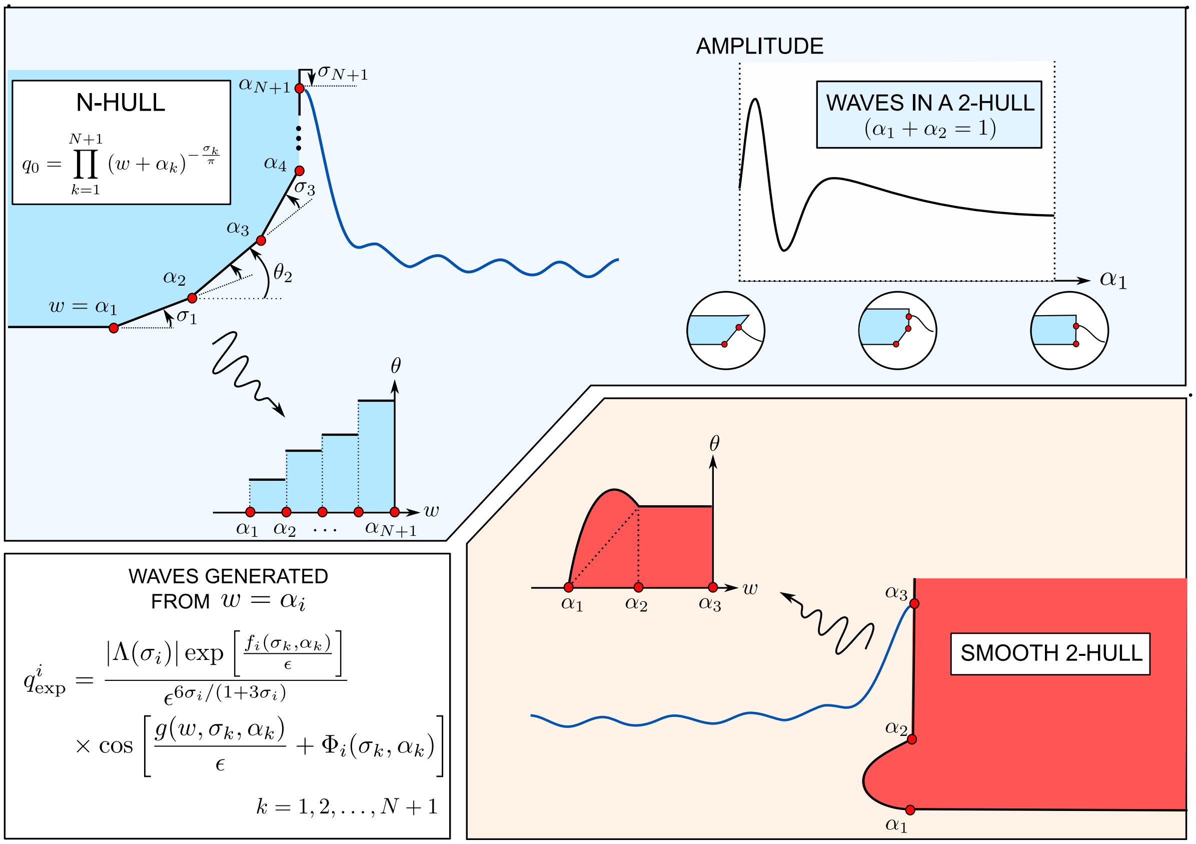

Figure 6: Exponential asymptotics can be applied to study the waves behind an *N-Hull*, a linear-piecewise ship with N corners (top left). If the contributing singularities are sufficiently widely spaced, each corner produces a wave with a given analytical form (bottom left); if not, then 'inner regions' with multiple singularities must be studied (top right). Research on smoothed hulls is ongoing (bottom right).

References

[1] Kruskal, M. and Segur, H. *Stud. Appl. Math.* **85**, 129–181 (1991).

[2] Chapman, S. *J. Fluid Mech.* **451**, 35–97 (2002).

[3] Witelski, T. and Bernoff, A. *Physica D* **147**, 155–176 (2000).

[4] Chapman, S. and Vanden-Broeck, J.-M. *SIAM J. Appl. Math.* **62 (6)**, 1872–1898 (2002).

[5] Ogilvie, T. Technical report, Michigan University, Ann Arbor, (1968).

[6] Dagan, G. and Tulin, M. *J. Fluid Mech.* **51 (3)**, 529–543 (1972).

[7] Vanden-Broeck, J.-M. and Tuck, E. In *2nd Internatinal Conference on Numerical Ship Hydrodynamics (Berkeley, Ca)*, (1977).

[8] Tuck, E. and Vanden-Broeck, J.-M. In *Proc. 15th Symp. Naval Hydrodynamics*, (1984).

[9] Farrow, D. and Tuck, E. *J. Austral. Math. Soc. Ser. B* **36**, 424–437 (1995).

[10] Baba, E. In *Proc. Int. Seminar on Wave Resistance, Tokyo*, 75–92, (1976).

[11] Madurasinghe, M. *J. Ship. Res.* **32**(3), 194–202 (1988).

[12] Tulin, M. *J. Ship Res.* **49**(4), 238–246 (2005).

[13] Chapman, S. and Vanden-Broeck, J.-M. *J. Fluid Mech.* **567**, 299–326 (2006).

[14] Rayleigh, L. *Proc. Lond. Math. Soc.* **15**, 69–78 (1883).

[15] Vanden-Broeck, J.-M. and Dias, F. *J. Fluid Mech.* **240**, 549–557 (1992).

[16] Dias, F., Menasce, D., and Vanden-Broeck, J.-M. *Eur. J. Mech. B/Fluids* **15 (1)**, 17–36 (1996).

[17] Forbes, L. *J. Fluid Mech.* **127**, 283–297 (1983).

[18] Grandison, S. and Vanden-Broeck, J.-M. *J. Eng. Math.* **54**, 89–97 (2006).

[19] Chester, W. and Bones, J. *Proc. R. Soc. Lond. A* **A306**, 23–39 (1968).

[20] Smyth, N. *J. Fluid Mech.* **192**, 287–312 (1988).

[21] Marchant, T. and Smyth, N. *J. Fluid Mech.* **221**, 263–287 (1990).

[22] Cox, E. and Mortell, M. *Z. Angew. Math. Phys.* **34**, 845–866 (1983).

[23] Cox, E. and Mortell, M. *J. Fluid Mech.* **162**, 99–116 (1986).

[24] Ockendon, J. and Ockendon, H. *J. Fluid Mech.* **59**, 397–413 (1973).

[25] Ockendon, J., Ockendon, H., and Johnson, A. *J. Fluid Mech.* **167**, 465–479 (1986).

[26] Amundsen, D., Cox, E., and Mortell, M. *Z. Angew. Math. Phys.* **58**(6), 1008–1034 (2007).

[27] Hirota, R. and Satsuma, J. *Phys. Lett. A* **89**, 168–171 (1981).