[MA20223 Vectors & PDEs] Exam study guide *Topic 1*

Last edited: 29 Mar 2019

Topic 1	Vector Calculus
Topic 2	Fourier Series
Topic 3	PDEs

Priority: All material on workbooks and in problem sets are examinable (except otherwise explicitly noted). All such concepts have a min. priority rating of 1.

Topic	Type	Can you...	Priority	Workbook chapter	Comment
		Chapter 1: The first one			
1	Derivation	Derive the transport equation	1	1	Not examinable though related to later discussion of deriving PDEs
		Chapter 2: Review on multivariable calculus			
1	Result	Explain the connections between normal vectors and gradients?	2	2	
1	Definition	Define conservative field and potential	3	2	
1	Method	Perform vector proofs with index notation	1	2	Index notation not examinable but can be used on exam to speed up any necessary proofs
		Chapter 3: Line integrals			
1	Definition	Define the vector equation of a curve; the tangent vector	3	3	
1	Definition	Define arclength in terms of a line integral	3	3	
1	Method	Parameterise different curves in 2D and 3D	3	3	Examples include: circles, y = f(x), ellipses, etc.
1	Definition	Define the line integral of a scalar field	3	3	
1	Definition	Define work integral	3	3	
1	Method	Compute different line and work integrals	3	3	PS1, Q1, Q5
1	Proof	Interpret the line and work integrals in terms of infinitessimal elements and forces	2	3	Related to the beginning lecture discussions; review notes and videos
		Chapter 4: The Big Theorem			
1	Discussion	Explain the intuition behind path independence of work integrals using the notion of gravity	1	4	
1	Theorem	State and prove FTC for work integrals	2	4	
1	Theorem	State and prove the big theorem on conservative forces	3	4	Theorem 4.3; PS1 Q4
1	Result	Explain why the simply-connected requirement is important in the definition of a conservative field	2	4	PS1, Q3
1	Result	Show that the work integral for a particle moving under F equals the change in kinetic energy	1	4	Example 4.4
		Chapter 5: Surface parameterisation			
1	Method	Compute explicit, implicit, and parametric representations of surfaces	3	5	Standard surfaces include: sphere, paraboloid, cylinder, cube, etc.; many examples, including PS2-4
1	Result	State the vector equation and normal to a plane	2	5	Lemma 5.3
1	Theorem	State and prove the formulae for the normal to a general surface (explicit and implicit)	2	5	Lemma 5.4
1	Method	Compute the normal to various surfaces	2	5	
		Chapter 6: Surface and flux integrals			
1	Starter discussion	Discuss why the surface integral formula is as it is (in terms of surface patches of area dS)	2	6	Starter discussion; PS2 Q6
1	Theorem	State the definition of a surface integral in terms of the explicit representation	3	6	Theorem 6.2; you should know the broad outlines of how it is proved.
1	Discussion	Explain what the connection is between the surface element dS and the Jacobian	1	6	Remark 6.3
1	Method	Compute various scalar surface integrals	2	6	E.g. Example 6.4 and problem sets
1	Definition	Define the flux integral	3	6	Definition 6.6

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Topic	Type	Can you...	Priority	Workbook chapter	Comment
1	Discussion	Interpret the flux integral in terms of flow through a surface	1	6	Lemma 6.8
1	Method	Compute flux integrals over different fields and surfaces	2	6	E.g. Example 6.7 and problem sets
1	Method	Derive different expressions for dS (dependent on chosen parameterisation)	2	6	PS2, Q2
		Chapter 7: Divergence and curl			
1	Definition	Define divergence and curl	3	7	
1	Definition	Define irrotational and incompressible	1	7	
1	Definition	Define the Laplace operator	3	7	
1	Method	Prove and compute various vector identites	2	7	PS3 Q2, Q4, Q5
		Chapter 8: Divergence and Green's theorem			
1	Starter discussion	Explain the intuition of the proof of the divergence theorem	1	8	This was done by explaining how to split the surface into little cuboids and calculating the flux through each cuboid
1	Theorem	State the divergence theorem	3	8	
1	Method	Apply the divergence theorem	3	8	Typical examples are "Verify the divergence theorem" types
1	Definition	Define the positive orientation of a curve	2	8	Definition 8.8
1	Theorem	State and prove the Stokes' theorem form of Green's theorem	2	8	Theorem 8.11; you do not have to state or prove the divergence form (Theorem 8.10)
1	Method	Apply Green's theorem to compute various quantities	2	8	E.g. Corollary 8.13 and problem sets
		Chapter 9: Stokes' theorem			
1	Starter discussion	Explain the basic idea of why Stokes' theorem is true	1	9	This was difficult!
1	Definition	Define what it means for a curve and surface to be "correspondingly orientated"	3	9	
1	Theorem	State Stokes' theorem	3	9	
1	Method	Apply Stokes' theorem	3	9	Lots of examples and problem sets
1	Theorem	Relate Stokes' theorem to conservative fields and irrotational forces	2	9	Theorem 9.3
		Problem sets			
1		Problem set 1: line integrals and conservative fields			Q1-5 relevant; Q6 can be used to teach yourself index notation (it helps, but not needed)
1		Problem set 2: surfaces and surface integrals			Everything relevant
1		Problem set 3: divergence, curl, and divergence theorem			Everything relevant
1		Problem set 4: Green's theorem and Stokes' theorem			Everything relevant

[MA20223 Vectors & PDEs] Exam study guide *Topic 2*

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Topic 1	Vector Calculus
Topic 2	Fourier Series
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Priority: All material on workbooks and in problem sets are examinable (except otherwise explicitly noted). All such concepts have a min. priority rating of 1.

Topic	Type	Can you...	Priority	Chap.	Comment
		Chapter 10: Introduction to PDEs			
3				10	Part of Topic 3
		Chapter 11: Introduction to Fourier series			
2	Starter discussion	Explain the basic idea of Fourier series	1	11	
2	Definition	Define the notion of a periodic extension; construct the periodic extension of a function.	2	11	This is done moreso by example; see Fig. 11.1 in the notes
2	Definition	Define even/odd functions	2	11	
2	Theorem	State/Prove the orthogonality properties of sines and cosines.	2	11	This is Lemma 11.8. You do not have to memorize product formulae (11.9).
2	Theorem	State/Prove the Fourier series formulae, including the coefficients an and bn, for a function defined on [-pi, pi].	3	11	This is Theorem 11.9. You should be able to prove this knowing Lemma 11.8.
2	Theorem	State/prove the expansion of f into a cosine or sine series.	3	11	This is Lemma 11.11.
2	Method	Find the Fourier series of various functions on [-pi, pi].	3	11	See problem sets and examples in lectures.
2	Method	Use Fourier series to establish formulae for infinite series.	2	11	Problem set questions
		Chapter 12: on Fourier convergence			
2	Starter discussion	Explain what "modes" and "spectrum" means; explain why convergence is a tricky aspect of Fourier series	1	12	
2	Method	Calculate the Fourier series of the square wave	3	12	Example 12.1
2	Remark	Discuss the Gibbs Phenomenon; discuss the nature of Fourier convergence	2	12	See end of chap. calculation of the Gibbs "overshoot"
2	Theorem	State the Fourier convergence theorem	3	12	Theorem 12.5
2	Theorem	State/prove Fourier series formulae for the case of extension from [-L, L].	3	12	Theorem 12.7
2	Definition	Define the even and odd periodic extension	3	12	Definition 12.9
2	Method	Given a function on [0, L], draw the periodic odd/even extension of that function.	3	12	
2	Method	Calculate the Fourier series for periodic extensions for f given from [0, 2L] or [-L, L]. Calculate Fourier series for even or odd extensions	3	12	Practice! Study all the basic examples like Fourier series of lines, parabolas, exponentials, constant functions, etc.
		Chapter 13: maths of music			
2					Not examinable! Chapter just for your enjoyment and appreciation of Fourier series
		Problem sets			
3		Problem set 5: introduction to PDEs			Part of Topic 3
2		Problem set 6: Fourier series I			Everything relevant
2		Problem set 7: Fourier series II			Everything relevant with exception of Q4 (priority 1)

[MA20223 Vectors & PDEs] Exam study guide *Topic 3*

Last edited: 16 Apr 2019

Topic 1	Vector Calculus
Topic 2	Fourier Series
Topic 3	PDEs

Priority: All material on workbooks and in problem sets are examinable (except otherwise explicitly noted). All such concepts have a min. priority rating of 1.

Topic	Type	Can you...	Priority	Workbook chapter	Comment
		Chapter 10: Introduction to PDEs			
3	Lemma	State the "bump" lemma, Lemma 10.1	1	10	You will not have to formally state this, nor prove it. But it is used everywhere in the all the derivation of the PDE models.
3	Proof	Derive the heat equation from physical principles in 1D	3	10	
3	Proof	Derive the wave equation from physical principles in 1D	3	10	
3	Proof	Derive the heat equation from physical principles in 3D	3	10	
		Chapter 14: Terminology of PDEs			
3	Definition	Define a Dirichlet condition in 1D/3D	2	14	This is largely subsumed by applications in Chaps. 15 onwards.
3	Definition	Define a Neumann condition in 1D/3D	2	14	
3	Definition	Define a Neumann condition in 1D/3D	2	14	
		Chapter 15: Separation of variables for the heat equation			
3	Method	Solve heat equation in 1D with zero [D] BCs	3	15	
3	Method	As above with zero [N] BCs	3	15	
3	Method	Solve for the steady-state heat equation in 1D	3	15	
3	Remark	Know what thermally insulated boundary conditions mean	3	15	
		Chapter 16: Separation of variables for the wave equation			
3	Method	Solve the wave equation in 1D with zero [D] BCs	3	16	
3	Method	As above with zero [N] BCs	3	16	
3	Remark	Plot individual modes for the wave equation	1	16	Example 16.2
		Chapter 17: Separation of variables for Laplace's equation			
		Not examinable			
		A word about this chapter: this chapter was removed from lectures and we covered a solution of the steady-state heat equation in a disc at the end of Chap. 15. This was done as well in PS8, Q4. This material allows you to link up with past exams, but it will not be examinable in 2018-2019.			
		Chapter 18: d'Alembert's formula			
3	Theorem	Show that the solution of the wave equation can be written as left and right-travelling waves	3	18	Theorem 18.1
3	Theorem	Prove d'Alembert's formula	3	18	Theorem 18.3
3	Method	Plot profiles of u(x,t) using d'Alembert's formula in the case g(s) = 0 by interpreting as left and right-travelling waves	3	18	Study the example on p. 180 of scanned notes; see 2017-2018 exam last question
		Chapter 19: Uniqueness			
3	Proof	Prove uniqueness for heat/wave/Poisson's equation in 1D with [D, N, M] conditions	3	19	
3	Proof	Same as above with 3D	3	19	All done PS10
		Problem sets			
3		Problem set 5: introduction to PDEs			Priority 3: Q1-3 Priority 2: Q4
3		Problem set 8: Heat equation			Priority 3: Q1 Priority 2: Q2, 3 Q4 can be ignored
3		Problem set 9: Wave equation			Priority 3: Q1, Q3 Priority 2: Q2 Priority 1: Q4
3		Problem set 10: Uniqueness			Everything Priority 3